Thrust Vector Control

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1 Introduction

This project looks to validate an airframe through the L2 certification process and in parallel develope thrust vector control.

2 Air Frame

The thrust vector control mission will utilize the same airframe used on the L2 mission [<**empty citation**>]. The aft section will be swapped with a new aft section which is upgraded to house the gimbal mount. This section will outline the design on the gimbal mount and the aft section which is compatible with the aft body tube coupler on the avionics bay. This compatablity will be described after the new designs.

2.1 Gimbal Mount

2.2 Aft Section

2.3 Full Assembly

3 Avionics

3.1 Hardware

The following items are installed in the avionics bay:

- CPU: Teensi 4.1 from XXX used for real time data processing.
- Intertial Measurement Unit (IMU): The BNO055 IMU that can measure acceleration, quaternion orientation, angular velocity rocket
- Altimeter: Measures the altitude of the rocket.
- Apple AirTag: Used to track the rocket after it has landed. One concern of the AirTag is the limitation of accelerations at launch. No official acceleration limit is provided in the air tag specification. So it is unknown if the AirTag will malfunction during launch acclerations.
- Micro SD card: used for logging flight data
- Batteries: power the CPU

3.2 Software

The software of the thrust vector control system was written in C++ and is formatted into a state machine. The flight computer receives input signals from the sensors described in the previous hardware section. For all states of flight, the output signal is run through a digital Butterworth Filter to provide a low-pass filter for smoothing sensor noise [1]. The states of the flight will be as follows:

1. Initial: The rocket has not taken off yet.
2. Ascent: The rocket motor has ignited and the thrust vectoring provides control.
3. Cruise: The motor has burned out but the rocket is still ascending.
4. Drogue Prachute: The rocket has reached apogee and deploys the drogue chute
5. Main Parachute: The rocket has descended to lower altitudes and the main chute is deployed.
6. Ground: The rocket has landed on the ground and power is conserved.

The following sections will derive each state of flight and how the code processes the data required transition to the next state.

3.2.1 Initial:

The state machine is initialized after the rocket has been set on the launch rail. This ensures that the rocket will not consume power by actuating the gimbal mount to attempt to correct the attitude with a non vertical launch rail. The state machine triggers to the accent state when magnitude of linear acceleration from the IMU is above $2\,m/s^2$.

3.2.2 Ascent:

Once ascent has been initiated, the gimbal can provide attitude control to the rocket as it is providing thrust. The state initiates its commands at 100Hz. Each initiation of the Ascent function, the physical motion and state of the rocket are computed. The acceleration and linear acceleration are provided in the body fixed frame of the IMU. This must be translated to the inertial frame by rotation of the orientation quaternion of the rocket. This is found by,

$$\begin{aligned} a_i &= q \cdot a_b \cdot q^* \\ a_{i,\text{lin}} &= q \cdot a_{b,\text{lin}} \cdot q^* \end{aligned} \tag{3.1}$$

where a_i and $a_{i,lin}$ are the total and linear accelerations, respectively, of the rocket in the inertial frame, a_b and $a_{b,lin}$ are the total and linear accelerations, respectively, of the rocket in the body fixed frame, and q is the orientation quaternion of the rocket.[1]

An approximation of the velocity in the inertial frame can be found by an Euler integration of the acceleration,

$$v_{k+1} = v_k + a_k \Delta t \tag{3.2}$$

This can be rotated back to the body fixed frame by,

$$v_b = q^* \cdot v_a \cdot q \tag{3.3}$$

The body fixed velocity can be used to find the drag on the rocket by,

$$D = \frac{1}{2} \rho v^2 C_d A \tag{3.4}$$

where C_d is the drag coefficient determined through CFD simulations, A is the frontal area of the rocket, ρ is the density of air by ideal gas law (I should check this because if the rocket is moving fast enough, ideal gas law may not hold true), and v is the velocity. (These are vectors and tensors that the bold font notation should be incorporated.)

By Newton's Second Law, the thrust can be determined by,

$$T + D + mg = ma \tag{3.5}$$

From manufacturer data on the motor (cite exact motor), the exit velocity exhaust remains sufficiently constant. This allows for a mass loss to be approximated throughout the ascent state. The mass loss is calculated by,

[1] The symbol $\otimes$ denotes Hamiltonian Quaternion multiplication.

$$\Delta m = \Delta t \cdot \frac{T}{v_e} \tag{3.6}$$

This provides a new mass of the rocket for the next timestep. This loss in mass will additionally change the mass moment of inertia on the rocket which is important for determining how much an impulse will effect the attitude of the rocket due to vectoring the motor. The change in mass moment of inertia about the axial axis will be negligible and the mass moment of inertia about the transverse axis will be approximated by,

$$\Delta I' = \Delta m \cdot d^2 \tag{3.7}$$

where d is defined by the distance between the rocket's center of mass and the motor's center of mass, which are both constantly changing. This change is approximated by...

The motor will provide torque with a moment arm of the distance from the motor nozzle to the center of mass of the rocket at a given timestep.

With the change in thrust, moment arm, and mass moment of inertia, it is desirable to have a dynamic coefficient for the PID weighting. This will be determined by the following,

$$K = \frac{I'}{T \cdot r} \tag{3.8}$$

This coefficient makes sense by lowering the weighting when I' is lower because it is easier to angularly accelerate the rocket. Additionally, the coefficient increases and T and r decrease because the vectoring provides less torque with less thrust and a smaller moment arm.

The next function that runs at each timestep of the ascent state is to undate the gimbal angle. This gimbal angle is determined by PID controls. The proportional term is determined by,

$$q_{error} = q^* \cdot q \tag{3.9}$$

This error angle is scaled by a factor, $K \cdot K_p$ where K is the physical coefficient previously calculated and K_p is an additional weighting that has been tuned for the proportional response.

The intergral term is a running tally of the proportional error during flight. It to correct for installation errors of the motor. This is calculated by,

$$q_k = q_{k-1} + q_{error} \tag{3.10}$$

This error angle is scaled by a factor, $K \cdot K_i$ where K is the physical coefficient previously calculated and K_i is an additional weighting that has been tuned for the integral response.

The derivative term is found so if the the rocket is moving towards the desired orientation, then the gimbal doesn't need to provide as much control. This is quantified in the controller by,

$$\dot{q} = \frac{1}{2} q \otimes \omega \tag{3.11}$$

Whether this $\dot{q}$ is moving towards or away from the desired orientation can be found by,

$$\dot{q}_{error} = q^* \cdot \dot{q} \tag{3.12}$$

This error angle is scaled by a factor, $K \cdot K_d$ where K is the physical coefficient previously calculated and K_d is an additional weighting that has been tuned for the derivative response.

Once the calculated thrust drops below 5 lbs, the gimbal mount is returned to the unit quaternion, $[1, 0, 0, 0]^\top$, and the state transitions to "CRUISE".

3.2.3 Cruise:

The cruise state is describes the phase of flight when the engine has burnt out, so no longer provides any control while vectoring the motor. However, the altitude is still increasing.

The state transitions to "DROGUE PARACHUTE" when apogee is hit. Appogee is detected by comparing the previous altimeter output to the current output. If the current altitude is less than the previous output nine times, then the state transition is triggered.

3.2.4 Drogue Parachute:

The drogue parachute state is implemented to delpoy the drogue parachute at apogee. Once the state transitions to drogue parachute, the forward ejection charge is ignited. After the drogue is deployed, the altitude is tracked at 10 Hz. Once the altitude is measured to be below 200 ft, the state transtions to "MAIN PARACHUTE".

3.2.5 Main Parachute:

The main parachute state is implemented to delpoy the main parachute at 200 ft. Once the state transitions to main parachute, the aft ejection charge is ignited. After the drogue is deployed, the altitude is tracked at 10 Hz. Once the altitude is measured to be below 2 ft, the state transitions to "GROUND".

3.2.6 Ground:

The ground state provides a condition for when the rocket has touched down. The power system should move to a state that ensures flight data is safe on the micro SD card, and power consumption is reduced.

4 Motor Selection

The motor for the thrust vector control mission must provide a long burn time while being compatible with the gimbal mount described in Section 2.1. The motor selected was the I59WN-P motor from Aerotech. This motor provides an average thrust of 59 N with a thrust duration of 7.99 seconds [<**empty citation**>]. The thrust curve of the motor can be seen in Figure 4.1

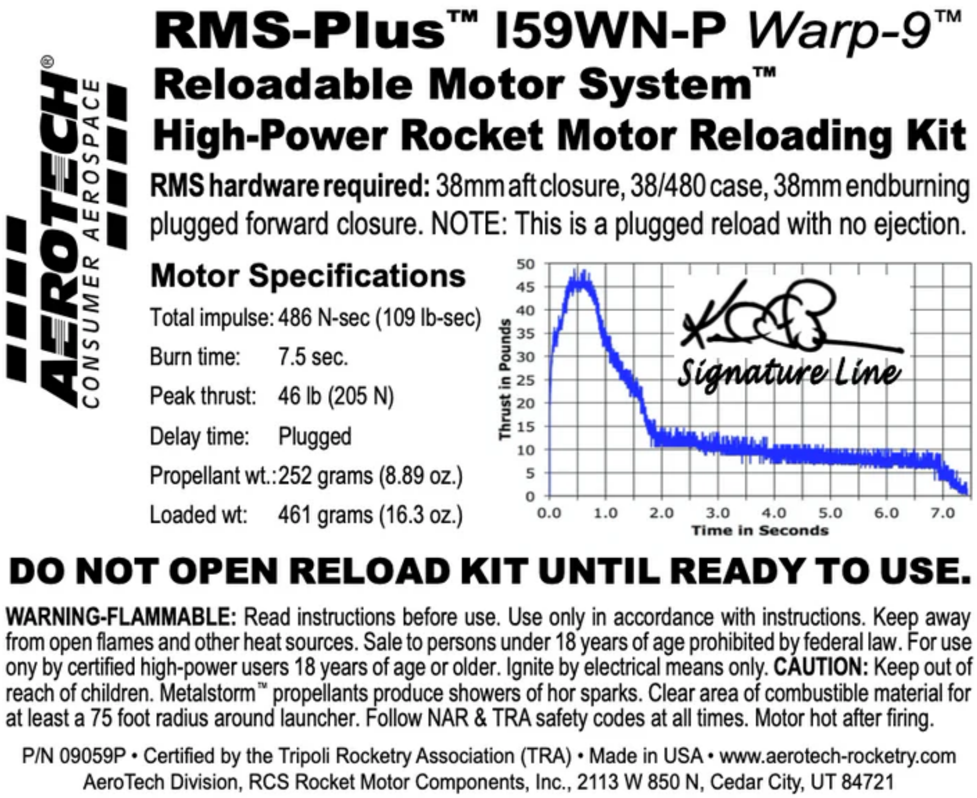

Figure 4.1: Motor specifications for I59WN-P from Aerotech [<**empty citation**>].

5 Simulink

The software developed in Section 3.2, was tested and tuned in a simulation using MATLAB Simulink. This simulation was designed to model the rigid body motion of the rocket through all stages of flight.

The simulation loop is built around three major subsystems, the rigid body dynamics, the noise generator, and the flight computer. These subsystems are shown in Figure 5.1.

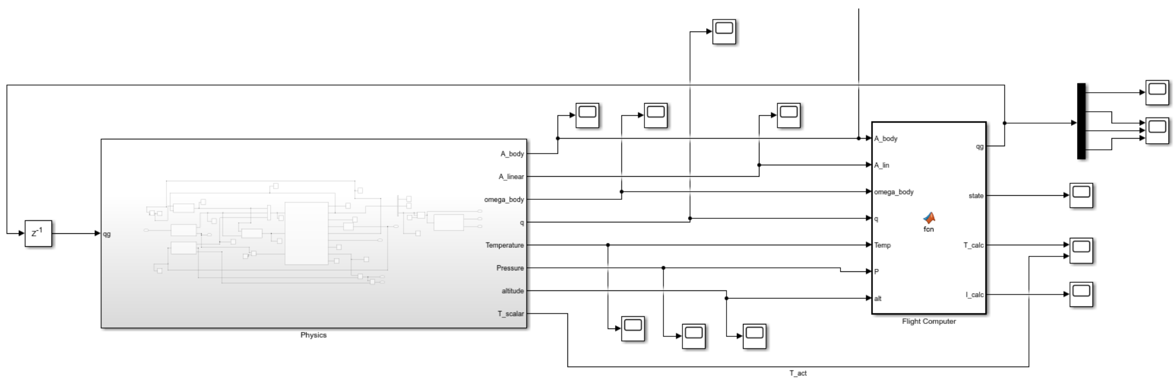

Figure 5.1: Caption

Expanding the rigid body dynamics subsection, it can be seen in Figure **??** that the dynamics are built around a quaternion, six degree of freedom block from the Aerospace Blockset [**aerospaceblock**].

The 6-DoF Quaternion block in the Aerospace Blockset numerically integrates the coupled translational and rotational equations of motion by propagating a state vector consisting of velocity, position, angular rate, and attitude quaternion through time.

$$m\,\dot{\mathbf{v}}_B = \mathbf{F}_{\text{ext},B} \tag{5.1}$$

In this translational equation, m is the vehicle's total mass and $\mathbf{v}_B$ its velocity expressed in the body-fixed frame. The term $\mathbf{F}_{\text{ext},B}$ represents the sum of all external forces—thrust, aerodynamic loads, and gravity—resolved into body coordinates.

$$\dot{\mathbf{p}}_I = R\big(q_I^B\big)\,\mathbf{v}_B \tag{5.2}$$

This kinematic relation maps body-frame velocity into inertial-frame position. Here, $\mathbf{p}_I$ is the inertial position vector and $R(q_I^B)$ is the direction cosine matrix derived from the quaternion q_I^B, which rotates body-frame vectors into the inertial reference frame.

$$I_B\,\dot{\boldsymbol{\omega}}_B + \boldsymbol{\omega}_B \times \big(I_B\,\boldsymbol{\omega}_B\big) = \mathbf{M}_{\text{ext},B} \tag{5.3}$$

Euler's equation of rotational motion appears here: I_B is the inertia tensor fixed in the body frame, and $\boldsymbol{\omega}_B$ is the body-frame angular velocity. The gyroscopic coupling $\boldsymbol{\omega}_B \times (I_B\,\boldsymbol{\omega}_B)$

is included, and $\mathbf{M}_{\text{ext},B}$ denotes the external moment vector (aerodynamic moments, control torques, etc.) in body coordinates.

$$\dot{q}_I^B = \tfrac{1}{2}\,\Omega\big(\boldsymbol{\omega}_B\big)\,q_I^B, \quad \Omega(\omega) = \begin{bmatrix} 0 & -\omega_x & -\omega_y & -\omega_z \\ \omega_x & 0 & \omega_z & -\omega_y \\ \omega_y & -\omega_z & 0 & \omega_x \\ \omega_z & \omega_y & -\omega_x & 0 \end{bmatrix} \tag{5.4}$$

Finally, the quaternion kinematic equation updates the attitude q_I^B based on the body-frame angular rate $\boldsymbol{\omega}_B = [\omega_x, \omega_y, \omega_z]^\mathsf{T}$. The matrix $\Omega(\omega)$ encodes the cross-product structure required to integrate quaternion attitude.

By propagating each of these four differential equations with Simulink's ODE solver (fixed- or variable-step), the block outputs the inertial position $\mathbf{p}_I$, inertial velocity $\mathbf{v}_I = R(q_I^B)\,\mathbf{v}_B$, attitude quaternion q_I^B (and optional Euler angles), and body-frame angular velocity $\boldsymbol{\omega}_B$, which feed directly into the flight computer subsystem to complete the closed-loop simulation.

The Force input of the 6-DoF block is defined by a sum of forces due to thrust, gravity, and drag. First, the thrust is resolved to a vector using the subsystem shown in Figure 5.2.

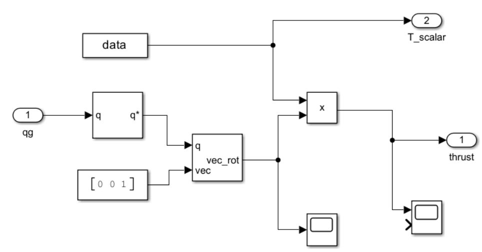

Figure 5.2: Caption

The thrust will act in the positive z-direction in the reference frame of the gimbal,

$$\mathbf{F}_{t,g} = [0, 0, T]^\top \tag{5.5}$$

where T is the thrust magnitude. This thrust magnitude is retrieved from an array in the workspace that give the thrust as a function of time which is based on data from the manufacturer of the I59 motor.

This thrust vector needs to be rotated into the body-fixed frame of the rocket. This is obtained through,

$$\mathbf{F}_{t,b} = q_g^* \otimes \mathbf{F}_{t,g} \otimes q_g \tag{5.6}$$

where q_g is the gimbal quaternion, q_g^* is it's conjugate, and $\mathbf{F}_{t,b}$ is the desired force vector due to thrust in the body-fixed frame.

The gravity force is resolved similarly, but the gravity vector is defined in the inertial frame by,

$$\mathbf{F}_{g,i} = [0, 0, m(t) \cdot g_h]^\top \tag{5.7}$$

where m is the mass of the rocket which is a function of time of the flight and g_h is the instantaneous acceleration due to gravity. The acceleration due to gravity in reality is decreasing as altitude increases, however to check the significance of this, the acceleration due to gravity is check at apogee, which is expected to be around 1km. This can be calculated by,

$$g_h = g\left(\frac{r_e}{r_e + h}\right)^2 \tag{5.8}$$

where g is the acceleration due to gravity at sea level, r_e is the radius of the Earth and h is the altitude [<**empty citation**>]. Evaluating g_h at an altitude of 1 km results in an acceleration of 9.804 m/s^2. This change will be considered negligible since it is less than a 0.1 percent change.

The change of mass of the rocket is resolved from the fact that the exit velocity remains relatively constant and the thrust is known, so by using,

$$\dot{m} = \frac{F_t}{v_e} \tag{5.9}$$

where ... $\dot{m}$ can then be subtracted from the total wet mass throughout the simulation. With the gravity defined in the inertial frame, this can be rotated to the body-fixed frame by,

$$\mathbf{F}_{g,b} = q \otimes \mathbf{F}_{g,i} \otimes q^* \tag{5.10}$$

where $\mathbf{F}_{g,b}$ is the force due to gravity in the body-fixed frame, q is the orientation of the rocket relative to vertical, and q^* is its quaternion conjugate.

6 References

[1] Bryce Quinton. *Butterworth Filter Report*. https://www.brycequinton.com/assets/pdf/Bryce_Quinton_Resume.pdf. Accessed: 2025-05-14. 2025.